Erotica, Aesthetics and Schrödinger's Wave Equation

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A good friend of Erwin Schrödinger recalled that 'he did his great work during a late erotic outburst in his life'. The epiphany occurred in the Christmas holidays of 1925, when the thirty-eight-year-old Viennese physicist vacationed with a former girlfriend at the Swiss ski resort of Arosa near Davos. Their passion was the catalyst for a year-long burst of creative activity. Like that of the dark lady who inspired Shakespeare's sonnets, her name remains a mystery, though most likely Schrödinger's wife was not in the dark about her husband's latest infidelity. Perhaps we owe to this unidentified woman the marvellous fact that apparently unconnected strands of research coalesced, and Schrödinger discovered the equation that bears his name.

In its form, at least, Schrödinger's equation was familiar to many scientists and its appearance was almost comforting in the light of the assault on familiar concepts coming from younger quantum physicists. It appeared to be the long-sought-after expression of the quantum theory that had been first articulated by its reluctant discoverer Max Planck in 1900, and had then been further refined by Albert Einstein and Niels Bohr, among others. In essence, it did for the subatomic world what Newton's laws did for the large-scale world some two centuries before – Schrödinger's equation enabled scientists to make detailed predictions about how matter behaves, while being able to visualize the atomic systems under study. Armed with the Schrödinger equation, it was possible for the first time to understand

atomic structure in detail, whereas Newton's equations simply did not make sense in the microworld.

A different quantum physics of the atom had been found some six months before Schrödinger's creative outburst. This was accomplished by Werner Heisenberg, a brilliant young German theoretician, working at the University of Göttingen. The twenty-four-year-old Heisenberg had discovered a distinctly different approach to atomic physics, couched in an unfamiliar and difficult mathematics that offered no scope for visualization of atomic processes and no equation analogous to Newton's for classical systems. In fact, one motive for Schrödinger's version of atomic physics was his distaste, amounting to disgust, for Heisenberg's. Schrödinger even proved the mathematical equivalence of the two versions – so which one was better? Schrödinger preferred his own and was adamant on this point. Heisenberg thought otherwise and immediately and forcefully staked his own claim.

But there is a paradox. Although the Schrödinger equation was superficially easy to use, it featured a quantity called the wave function that was extremely difficult to interpret and impossible to observe directly. Heisenberg vehemently disagreed with Schrödinger's interpretation of the wave function as representing an atomic electron's smear of electricity around the nucleus. An enormous and bitter controversy arose that is not fully resolved even today. Schrödinger himself was never happy with the most common interpretation of the wave function's meaning.

In this essay, I want to explore how Heisenberg's *interpretation* came to dominate over Schrödinger's, even though Schrödinger's *method*, suitably reinterpreted, replaced Heisenberg's in almost all areas of physical theory. The issues under intense debate at that time – how we are to visualize atomic behaviour, and whether it is purely understandable in terms of probability – still echo through physics today. From a practical point of view, however, quantum theory has proved enormously successful. It has formed the basis of our understanding of the microworld, enabling technologists to develop increasingly effective transistors, microprocessors, lasers and fibre-optic cables. The theory is mainly implemented through Schrödinger's equation, which is used as a routine research tool by scientists all over the world.

Born in Vienna in 1887, the cultural and political capital of the Austro-Hungarian Empire, Schrödinger attended a gymnasium, or high school, which emphasized the study of Greek and Latin classics. Schrödinger also taught himself English and French.[1] He excelled at school and was recognized as a student of genius calibre. This wide and deep education served to ingrain in him a profound respect for classical tradition. His book *Nature and the Greeks*, published in 1948, is an elegant exposition of ancient physical theories and their relevance. Schrödinger developed a lifelong interest in philosophy, which would lead to more than casual readings in Eastern texts such as the Vedanta, which he wrote about in 1925 in an intensely personal account of his beliefs, *Seek for the Road*. It is influenced by Hinduism, and is an argument for the essential oneness of human consciousness and for the unity of humanity and nature. This was not published until 1961, the year before his death, as part of the volume called *My View of the World*.

Although Heisenberg also attended a gymnasium and had a flair for music and philosophy, his mind-set differed radically from that of the more conservative Schrödinger, his senior by a decade and a half.[2] Heisenberg revelled in situations that were in flux. It cannot be irrelevant that he came of age in one of the most turbulent periods in German history, amid defeat in World War I, the collapse of the monarchy and revolution spreading across the Reich. Like Schrödinger, Heisenberg came from a cultured family; he was a pianist of near-concert level. Music was central to his life, while Schrödinger had no feeling for it. They were united, however, by their vigour and youthfulness, which both men sustained to an advanced age.

In 1906 Schrödinger entered the University of Vienna, where he had excellent teachers. He flourished in this atmosphere, deepening his understanding of physics and adding to it an interest in biology, to which, some forty years later, he contributed some profound ideas in his short book *What is Life?* (James Watson, co-discoverer of the structure of DNA, cited it as an inspiration.)

By this time Schrödinger's highly developed erotic instinct had begun to emerge. It differed in his mind from the traditional male-chauvinist goal of female domination. Rather, Schrödinger believed he was exploring the essence of female sensuality. And he kept a logbook with comments, dates and names of encounters, his *Ephemeridae*. Like the then avant-garde artist Gustav Klimt, Schrödinger forever sought 'to capture the feeling of femaleness'. We can imagine that Schrödinger's calculated casualness of dress and appearance, with his high forehead, carefully combed hair and intense gaze, in conjunction with a seemingly inexhaustible well of knowledge, was very attractive to women. Despite his bourgeois demeanour and correctness, there was something Byronic about Schrödinger.

Like such Viennese compatriots as Ludwig Wittgenstein, he played an active role in World War I, serving with distinction on the Italian front in an artillery unit of the Austro-Hungarian army. Schrödinger was cited for his leadership in the face of fierce counter-battery fire in October 1915 in the course of the bloody battles made famous by Ernest Hemingway in *A Farewell to Arms*. Soon after, he was promoted to *Oberleutnant* and finished the war in Vienna in the cushy position of teaching introductory meteorology to army officers, while publishing papers on gas theory and general relativity.

By 1925 Schrödinger differed in every way from the brash younger men bursting onto the scene in quantum physics. Even his dapper style of dress contrasted with Heisenberg's, who is remembered as looking 'like a simple farm boy, with short, fair hair, clear bright eyes, and a charming expression'.[3] Compared with Heisenberg and his colleague and confidant, the hypercritical and acerbic Wolfgang Pauli, Schrödinger was already a senior figure, with a professorship at the University of Zurich.

Heisenberg's undergraduate education was extraordinary. Fortuitously, when he entered the University of Munich, during the winter semester of 1920–21, the famous physicist Arnold Sommerfeld was about to teach the atomic-physics part of his theoretical-physics cycle. In this way Heisenberg was thrown right onto the cutting edge of research. He often recalled that he learned physics backwards, studying atomic physics before Newton's, which is supposed to be the stepping stone to advanced topics.

The atomic theory current at the time was formulated in 1913 by the twenty-seven-year-old Danish physicist Niels Bohr. Bohr's was a frighteningly intense search for clarity, in a lifelong journey that he would share with colleagues and students in deep, critical dialogues. For this reason many of Bohr's scientific papers are almost opaque, having been worked and reworked so many times that omitting a word or a sentence can completely distort its meaning. He thought at his best in conversation, when he had someone against whom to bounce ideas. But in 1913 he was a young man in a hurry and moved with a gracefulness honed by the high level of football that he played. Ten years later he would begin to assume the heavy, gloomy veneer that reflected the weight of the problems he took upon himself in seeking the meaning of a new physics, one that defied all preconceptions of what a theory ought to be.[4]

Bohr's atomic theory of 1913 is best remembered for its hallmark imagery of atoms as minuscule solar systems. It was a magnificent pastiche of Newton's celestial mechanics with adroit insertions from Planck's radiation theory. Bohr's use of Newton's theory permitted its imagery to be transported into the atomic realm. This enabled him to restrict the electrons bound in atoms, that is, atomic electrons, to certain orbits about their central sun, or nucleus. These allowed orbits are called stationary states or energy levels. Consider the hydrogen atom, which is the simplest atom because it comprises a single electron that is bound to a positively charged nucleus. According to Bohr's theory, this electron can exist only in certain orbits. The lowest allowed orbit – the one closest to the nucleus – is referred to as the atom's ground state. An astounding consequence of Bohr's theory is that while in an allowed state the electron just perches, like a bird in a tree, doing nothing but waiting. By contrast, according to the accepted electromagnetic theory of the day, combined with Newton's mechanics, the electron ought to be orbiting the nucleus like a planet around the sun. According to the traditional laws of physics, the orbiting electron would continuously give off radiation energy. Consequently the atomic electron would lose energy and eventually spiral into the nucleus. The result is that matter would be totally unstable. We know this is not the case because, for example, you are sitting here and reading this article, instead of exploding. Explaining atomic stability was considered to be a key problem at the time. Bohr, however, had the great creative insight to realize that it was for the present insoluble and so just accepted it as a fact of life. This was his reason for postulating the existence of a lowest stationary state, or orbit, in which the electron neither drops nor radiates any light – and no further questions, thank you.

By, for example, illuminating the atom with light it is possible to excite the electron into a higher allowed orbit. Once there the electron is again like a perching bird, only now waiting to descend back to the ground state. Eventually it will come down, either directly or perhaps by transitions between states above the ground state. These transitions are not smooth but discontinuous and so are called quantum jumps. It is while making such transitions between stationary states that the electron emits radiation in bursts – that is, discontinuously. An enormous success of Bohr's theory was its ability to account for the wavelengths of the radiation emitted by hydrogen to within 1 per cent of the values experimenters had observed. Moreover, it successfully predicted previously unobserved wavelengths to a similarly high accuracy.

Bohr's theory caused great excitement in the physics community. One eminent and very sober physicist of whom more later – Max Born – said of Bohr's theory that it performed 'a great magic on mankind's mind; indeed its form is rooted in the superstition (which is as old as the history of

thought) that the destiny of men could be read from the stars'. Einstein immediately praised the theory as 'an enormous achievement'.

By early 1925, however, the situation in atomic physics had become direly confused. The consensus among physicists was that Bohr's atomic theory was at a dead end. It could not treat with any accuracy anything except simple instances of the hydrogen atom. By 1923 data began to accrue from the interaction of atoms with light, to the effect that atoms did not respond like minuscule solar systems after all.

Physicists quickly cobbled together a hybrid version of Bohr's theory and this served as a stopgap. In this lash-up, no attempt whatever was made to visualize what was going on at a subatomic level but it was assumed that atoms could somehow lose energy by making a transition between one energy level and a comparatively lower one – by making a 'quantum jump'. Likewise, the atom could gain energy by jumping between energy levels to a higher one. In each of these processes, the energy lost or gained is carried by a burst of light corresponding to radiation with a particular wavelength. This explains why atoms emit and absorb radiation with special wavelengths, known as spectral lines. Another crucial feature of this unsatisfactory theory was the innovative idea that it was not possible to predict exactly when atoms made quantum jumps – it was possible only to quote the probability of such an event's taking place at a particular instant. Bohr imported this 'probabilism', which came to be a central feature of quantum thinking, from a successful theory Einstein had introduced in 1916, in his theory about the interaction of radiation and atoms. These three features of the improved quantum theory of the atom – probabilism, quantum jumps and non-visualizability – were sufficient to make the theory serviceable until the beginning of 1925, when it, too, folded.

Physicists interpreted probabilities as a sign of not truly understanding the mechanisms of individual processes. They believed that eventually the mechanism by which electrons made transitions in atoms would come to be understood, and some as yet unknown version of Newton's mechanics would be formulated. In the end it would be business as usual, and probabilities would be unnecessary. This would turn out not to be the case. Although the modified version of Bohr's theory ultimately failed, it served as Heisenberg's stepping stone to his dramatically new atomic theory which was based on unvisualizable electrons and radical discontinuities. Its foundation was a mathematics that posed extreme difficulties in actually applying it. Heisenberg himself, in his first paper on the quantum mechanics, did not understand how to use it.

He had stumbled across mathematical quantities called matrices. This is because Heisenberg was interested in finding a sort of bookkeeping method for all possible atomic transitions between stationary states. Matrices are a natural way to do this, in addition to providing machinery for calculating the characteristics of spectral lines. To be a bit more precise: matrices are square arrays of numbers, and in quantum mechanics each entry represents a possible atomic transition, either up or down in energy. Through a well-known mathematical method the energies of the atom can be calculated. These are called the matrices' proper values, or eigenvalues, and their calculation is usually arduous. Wolfgang Pauli, one of the strongest calculators of the day, took over forty pages to deduce the energy levels of the simple hydrogen atom from Heisenberg's theory. By the end of 1925, certain long-standing problems had been solved by Heisenberg and his co-workers, which had eluded Bohr's theory. Heisenberg's 'matrices' version of quantum mechanics seemed to promise a great deal.

Heisenberg's hybrid education was undoubtedly one of the sources of his daring, hell-for-leather approach to atomic-physics research. Less than a year after entering university, he wrote his first paper, in which he chose not to work from certain rules for translating results from Newton's physics into quantum physics, as was the accepted method, but from a model already somewhat in agreement with quantum ideas. As one of Heisenberg's colleagues said later, 'A wonderful combination of profound intuition and formal virtuosity inspired Heisenberg to conceptions of striking brilliance.'

At this time Schrödinger was pursuing, as usual, a wide variety of interests. Besides investigations in general relativity, since 1917 he had been studying the perception of colours. Then there was his interest in problems concerning sound and elastic media, which led to his investigation of wave theory, soon to come in handy.

On the personal side, Schrödinger was living in Zurich with his wife of some five years, Annemarie, known affectionately as Anny. They lived in a slightly subdued north Swiss version of the bohemian Weimar culture that so scandalized German conservatives and nationalists: the sexually shocking, ambiguous world we associate with Marlene Dietrich and expressionist art and cinema. The violent reaction to this liberated milieu was personified by Hitler and his Nazi Party; the rise of such thuggish violence would give Schrödinger second thoughts about leaving Zurich for Berlin in 1927 to take Planck's chair. Meanwhile, the marriage was clouded on the one hand by

Anny's never conceiving the child that Erwin so badly wanted and on the other by his compulsive womanizing. They were an odd couple. Anny had limited intellectual interests and worshipped Erwin's looks and brilliance. After their passion waned – about a year into their marriage – they both sought sex elsewhere, and yet they remained married and cared for each other as friends. As Anny commented some years later, 'You know it would be easier to live with a canary than a racehorse, but I prefer the racehorse.' Schrödinger never had a close male friend in his entire life. His flair for dress and his romantic intensity towards women were fuelled by his love for the theatrical.

From considerations based on relativity theory, in 1923 Louis de Broglie suggested that electrons can also be waves, whereas previously everyone thought of them exclusively as particles. At once Einstein recognized the importance of de Broglie's observations and elaborated on them in research on gas theory. Einstein was enthusiastic and wrote to a colleague that de Broglie 'has lifted a corner of the great veil'. But de Broglie and Einstein were only part of the impetus behind Schrödinger's orgy of creativity, as he explains in the third of the papers he published in the spring of 1926:

> My Theory was inspired by L. De Broglie, *Ann. de Physique* (10) 3, p. 22, 1925 (Thèses, Paris, 1924) and by short but incomplete remarks by A. Einstein, *Berl. Ber.* (1925) pp. 9 ff. No genetic relationship whatever with Heisenberg is known to me. I knew of his theory, of course, but felt discouraged, not to say repelled, by the methods of transcendental algebra, which appeared very difficult to me and by the lack of visualizability.[5]

The sense of aesthetics that Schrödinger alludes to here is his preference for a mathematics that is more familiar and also not as ugly as Heisenberg's 'transcendental algebra' (or matrices), but which also permits visualizability of atomic processes. This becomes clearer in what follows.

In a more objective tone, one of Schrödinger's principal criticisms of Heisenberg's quantum mechanics is that it appeared to him 'extraordinarily difficult' to approach such processes as collision phenomena from the viewpoint of a 'theory of knowledge' in which we 'suppress intuition and operate only with abstract concepts such as transition probabilities, energy levels, and the like'. Indeed, in Heisenberg's formulation during 1925–26 it was possible only to calculate atomic energy levels; that is, to deal only with electrons bound in atoms. On the other hand, the concept of what is abstract is relative: Bohr, Heisenberg and Pauli considered energy levels 'and the like' to be perfectly concrete. Schrödinger admitted in 1926 that there may exist 'things' that cannot be comprehended by our 'forms of thought', and hence do not have a familiar Newtonian space and time description, but 'from the philosophic point of view' he was sure that 'the structure of the atom' does not belong to this set of things.[6]

But Schrödinger did realize, along with Heisenberg and other physicists of the time, that visual imagery taken wholesale from the world of sense perception would not suffice. In order to avoid it altogether, Heisenberg based his quantum mechanics on unvisualizable particles. Schrödinger sought a means of visualizing electrons that was different from the way in which scientists had become accustomed to thinking of them; that is, as particles. He realized that such an approach to the electron had been made available by de Broglie and Schrödinger and set out to exploit it. This may have been an aesthetic preference, but it was one on which theories could be based. Building on de Broglie's daring idea that electrons can be waves as well as particles, Schrödinger applied this hypothesis to electrons bound in atoms.

Schrödinger's basic idea was to formulate a theory for electrons bound in atoms in which they are analogous to a vibrating string fixed at both ends. How the string vibrates is an indicator of the electron's energy. This sort of wave theory also avoids quantum jumps. The reason is that atomic transitions were understood as occurring in the manner of waves representing the electron's charge density, surrounding the nucleus and decreasing their radius in order to pass between allowed states.

Let me describe how Schrödinger applied his ideas to the simplest atom of all, a single electron orbiting a nucleus – a hydrogen atom. As a thought experiment, consider the electron as a string fixed at both ends; that is, bound in a hydrogen atom. When the string is vibrating at its lowest energy as a standing wave, there is exactly one half-wavelength between the ends. At the next highest energy, there are two half-wavelengths between the ends; then, at the next highest energy, three half-wavelengths, and so on. The point is that each configuration of the vibrating string corresponds to a particular energy, or eigenvalue, of the string.

Schrödinger's equation, when applied to the hydrogen atom, yielded much the same relationship between energy levels and allowed wave functions. The equation predicts the possible energy values that the electron can

have (its energy levels, each denoted by E) along with the so-called wave functions that describe its behaviour (each of them is denoted mathematically by the Greek letter ψ, psi). The equation says

$$\hat{H}\psi = E\psi$$

The letter $\hat{H}$ stands for the mathematical expression (known technically as an operator) that represents the total energy of the atom. After the mathematics has been done, one ends up with a set of energy levels, each with at least one corresponding wave function.[7, 8]

The amazing thing was that this simple mathematical operation predicted exactly the right energy levels for the hydrogen atom, reproducing the success of Bohr's planetary model. But how should we picture orbiting atomic electrons in Schrödinger's picture? That's where it gets difficult. Schrödinger visualized the atomic electrons as a distribution of electronic charge whose distribution in space is related to the electrons' wave function.

Great though Schrödinger's achievement was in writing down his equation, it is at odds with the special theory of relativity. Basically the equation is inconsistent with guidelines set down by Einstein's principle of relativity, according to which an equation should have the proper mathematical form so as to be able to include measurements made on systems moving at high speeds close to that of light. But this was intentionally so because Schrödinger initially tried a relativistic approach and failed. What he did was to insert de Broglie's results into the relativistic equation connecting energy, momentum and mass. Then he specialized to that benchmark for all quantum theories, the hydrogen atom, in order to calculate its spectrum of energy values. He failed because Schrödinger's relativistic equation did not include the electron's spin, a property that was only beginning to be understood at the time. On the other hand, Schrödinger found that a non-relativistic version gave results in agreement with observation. This problem would be overcome in 1928 when the English theoretician Paul Dirac brilliantly proposed a quantum equation for the behaviour of the electron that was consistent with special relativity. This equation naturally explained why the electron has spin.[9] Looking back on this episode, Dirac wrote that Schrödinger should have pursued the relativistic equation because, in his view, 'It is more important to have beauty in one's equations than to have them fit experiment.'[10]

How did Schrödinger derive his equation? The derivations that Schrödinger presented in his published papers for the Schrödinger equation are in fact not derivations at all, but plausibility arguments: he knew beforehand what he wanted. Actually the Schrödinger equation should be considered axiomatic, that is, underivable: its validity comes from the correct solutions it gives to certain problems such as the hydrogen atom spectrum which Schrödinger disposed of in a few pages, compared with Pauli's mathematical gymnastics using Heisenberg's quantum mechanics.

Schrödinger went on to prove the mathematical equivalence of the wave and quantum mechanics and pushed this result to support his disdain for quantum mechanics: when discussing atomic theories he 'could properly use the singular'.[11] For Schrödinger, *sic transit* quantum mechanics. But what sort of picture did Schrödinger offer? He maintains that no picture at all is preferable to the miniature solar-system atom, and in this sense the quantum mechanics is preferable because of 'its complete lack of visualisation'; however, this conflicts with Schrödinger's philosophical viewpoint. Schrödinger argued that the wave function for, say, the electron in the hydrogen atom is related to the electron's distribution of electricity around the nucleus. However, Schrödinger's proof of the localization of the waves representing the electron turned out to be incorrect, as Heisenberg showed in 1927: the waves representing the electron do not in general remain localized, that is, stay together.[12] But Schrödinger was intellectually honest in emphasizing that his claimed visual representation is unsuitable for systems containing more than one electron. The reason is that the wave function representing a single electron can be visualized as a wave in three dimensions because it depends on the electron's position in a three-dimensional space. The wave function for a system of two electrons depends on both of their spatial positions, and so is three plus three or six-dimensional, whereas our visual perception is restricted to three dimensions.

The state of quantum mechanics in the first half of 1926 can be summarized as follows. No adequate atomic theory had existed as of mid-June 1925, but by mid-1926 there were two seemingly dissimilar theories. Although a particle-based theory, Heisenberg's renounced any visualization of the bound particle itself, its mathematical apparatus was unfamiliar to physicists and difficult to apply, and it was based specifically on discontinuities. But discontinuity is anathema to Newtonian physics as well as to the pre-quantum version of electromagnetism in which all processes occur continuously and are visualized as waves. On the other hand, Schrödinger's wave mechanics focused upon *matter* as waves, offered a visual representation of atomic phenomena (albeit restricted to a single electron), and served to account for discrete spectral lines without quantum jumps. The

Schrödinger theory's more familiar mathematical apparatus of differential equations set the stage for a calculational breakthrough, supported by Schrödinger's proof of the mathematical equivalence of the two theories.[13] Wave mechanics delighted the portion of the physics community that resisted discontinuity being built into physics and preferred a version of atomic physics based on a theory similar to Newton's. Although conclusive evidence of the wave-particle duality for electrons would not appear until 1927, experiments performed as early as 1923 agreed with de Broglie's hypothesis. Consequently many physicists tended to accept it. As Einstein wrote to Schrödinger on 26 April 1926, 'I am convinced that you have made a decisive advance . . . just as I am equally convinced that the Heisenberg . . . route is off the track.'

Heisenberg's first recorded comment on Schrödinger's wave mechanics is in a letter of 8 June 1926 to his friend and colleague Pauli, and he was enraged: 'The more I reflect on the physical portion of Schrödinger's theory the more disgusting I find it. What Schrödinger writes on the visualizability of his theory is probably not quite right. In other words it's crap.'

During this troubled period in Heisenberg's professional life, he was most candid with Pauli, then at the University of Hamburg. Pauli's interests were always wide-ranging, and included such esoterica as numerology and the Kabbala. Nor was he averse to dipping into the Hamburg underworld of drugs and sex. In the early 1930s Pauli became a devotee of the Zurich psychoanalyst Carl Jung. The two went on to co-author a book in which Pauli wrote a memorable Jungian analysis of the great astronomer Johannes Kepler, a man not unlike Pauli in his extra-scientific interests.

In addition to sharp comments in letters to Pauli, Heisenberg responded quickly in print as well, although in a more piano tone. In a paper of June 1926 he wrote that although the physical interpretations of the two theories differ, their mathematical equivalence allows this difference to be put aside; for 'expediency' in calculations he will utilize Schrödinger's wave functions, with the caveat that one must not impose upon the quantum theory Schrödinger's 'intuitive pictures'.[14]

Schrödinger and Heisenberg first encountered each other in July 1926 in Munich, where Arnold Sommerfeld had invited Schrödinger to deliver two lectures on his new theory. There was standing room only. Barely keeping himself in check, after the second lecture Heisenberg rose to deliver what was essentially an impromptu monologue attacking Schrödinger's wave mechanics because it was apparently unable to explain how radiation interacts with matter through quantum jumps. Amid shouts of disagreement from the audience, the angry chairman, an eminent Munich physicist, motioned to Heisenberg to sit down and be quiet. Later on, he told Heisenberg that his physics 'and with it all such nonsense as quantum jumps [is] finished'. Heisenberg was despondent because he seemed to be unable to convince anyone of his views. But he continued to argue his case, and by August 1926 colleagues began to write worried letters to Schrödinger asking how indeed he could explain certain quantum effects without discontinuities. Schrödinger himself began to feel uncertain.

Tension between quantum mechanics and wave mechanics increased with the publication of results by Heisenberg's mentor at the University of Göttingen, Max Born, in July 1926. (Max Born later became a footnote in pop music as the maternal grandfather of Olivia Newton-John.) The forty-five-year-old Born was a rather shy, withdrawn character who directed one of the three institutes where Heisenberg studied. (The other two were Sommerfeld's at the University of Munich and Bohr's in Copenhagen.) Heisenberg had discovered his quantum mechanics while spending a period of time away from Munich at Göttingen. At Born's institute, physicists were interested in exploring the nature of electrons as particles by arranging for them to hit and scatter off atoms. This is a very different sort of physics problem from dealing with electrons bound in atoms. Born was interested in 'free' electrons, that is, electrons that have no net force acting on them. But at this time neither quantum mechanics nor wave mechanics could deal with free electrons, as they move through space.

Originally trained as a mathematician, Born had quickly grasped the subtleties in the Heisenberg and Schrödinger formulations in addition to their physical content. So everyone listened when Born wrote of the deficiencies of both men's theories in accounting for scattering experiments. And so Born decided that 'new concepts' were needed and wave mechanics would be his vehicle, because at least it presented the possibility of some sort of visual imagery.

Born made the stunning proposal that Schrödinger's wave function represents neither the electron's visualizable charge distribution as a wave surrounding the atom's nucleus, nor a group of charge waves moving through space. Rather the wave function is a totally abstract quantity in that it is not at all amenable to any visualization. Instead of being able to calculate from it a density of electricity, one calculates something that acts like a density – a probability density for the electron to be present in some region of space. This dramatic assumption transformed Schrödinger's equation into a radically new form, never before contemplated. Whereas Newton's

equation of motion yields the spatial position of a system at any time, Schrödinger's produces a wave function from which a probability can easily be calculated. Schrödinger's equation then tells us not the path of a particle, but how the probability of the particle's detection changes with time. Born's aim was nothing less striking than to associate Schrödinger's wave function with the presence of matter.

By the autumn of 1926 Heisenberg had come to hate Schrödinger not only because his equation was so widely used – professional jealousy should never be discounted among creative people – but also for another, no less important reason, one that impacted directly on the very depths of Heisenberg's own research programme. He recalled, 'Schrödinger tried to push us back into a language in which we had to describe nature by "intuitive methods". That I couldn't believe. That is why I was so upset about the Schrödinger development in spite of its enormous successes.[15] After all, Schrödinger's equation was incredibly simpler to use than the mathematics in Heisenberg's quantum mechanics.' Then came Born's paper in which 'he went over to the Schrödinger theory'. Heisenberg described these developments as very disturbing to his 'actual psychological situation at that time'.

In November 1926 Heisenberg published a paper that got very little attention but, he recalled, 'for myself it was a very important paper'.[16] It was written by an angry man, and in it Born's scattering theory is nowhere cited and Schrödinger is sharply criticized. Heisenberg demonstrates that a probabilistic interpretation can be understood only if there are quantum jumps; that is, discontinuities. The gist of Heisenberg's paper is to prove that the presence of probabilities implies discontinuous phenomena, which in turn requires the presence of particles that are, after all, discontinuities in the fabric of nature. And so he comes down firmly in favour only of a particle viewpoint and, by implication, against Schrödinger's wave mechanics.

In subsequent articles during that year Heisenberg emphasized that phenomena occurring in the small volumes of the subatomic contradict our customary intuition. By this he meant that, contrary to Schrödinger, terms derived from day-to-day understanding of the world like 'wave' and 'particle' cannot be glibly extended into the atomic world. Surprises lurk there such as the wave-particle duality of light, first made explicit by Einstein in 1909, and the wave-particle duality for electrons proposed by de Broglie in 1923. This dual mode of existence is totally counter-intuitive and unimaginable. How can something be continuous and discontinuous at the same time? For this reason, physicists were slow to accept Einstein's light quantum. Their principal reason, stated forthrightly by Planck in 1910, was that when light was shone on alternating strips of opaque and transparent material (known as a diffraction grating) it behaves like water waves, producing a smoothly varying pattern of light that cannot be explained by assuming that light behaves as particles. This profound problem was solved only in 1927 when Born put forward his interpretation of the wave function, which accounted for these diffraction patterns in terms of myriad tiny impacts by individual particles of light. For many physicists, however, the mixture of wave and particle elements in the explanation remained deeply puzzling.

Just as a particle representation for light seemed out of place, so, at first, did a wave representation for the electron, as it was proposed in 1923 by de Broglie. Physicists were eventually persuaded to accept a wave-particle duality for the electron, because experimental data in that year lent some support to de Broglie's hypothesis. Conclusive results were achieved in 1927. Yet evidence for the existence of light quanta appeared in experiments of 1923. But even the person who performed these experiments, Arthur Compton, couldn't believe the results. His principal objection lay in the relation between the energy of the light quantum (which is, after all, a particle, and is therefore localized) and its wavelength (which is not localized). How can such entirely different quantities be related at all? Is this not like trying to link fishes and rocks? Clearly the wave nature of electrons, which would by 1927 be accepted as conclusive, did not upset physicists as much as disturbing the centuries-old sacred representation of light as a wave.

To Heisenberg, as to Schrödinger, the fundamental issue in quantum theory became, as Heisenberg put it, to explore the 'kind of reality' that existed in the atomic world. Physics had become a branch of metaphysics because nothing less was at stake than understanding the nature of physical reality. Heisenberg took on this problem in his classic paper of 1927, 'On the intuitive content of the quantum-theoretical kinematics and mechanics', the so-called 'uncertainty principle' paper.[17] The term 'intuitive' in the title signals that this absolutely fundamental concept has to be redefined in the atomic world. Straightaway Heisenberg makes it clear that the basic issue facing quantum mechanics is the meaning of certain terms when they are extrapolated into the atomic realm: 'The present paper sets up exact definitions of the words: position, velocity, energy, etc. (e.g. of an electron).' Heisenberg insists that it is the *interpretation* of quantum mechanics that is in question: 'Heretofore, the intuitive interpretation of the quantum mechanics is full of internal contradictions that become apparent in the struggle of the opinions concerning discontinuum- and continuum-theory, wave and particles.' He reasoned that a new intuitive interpretation, replete with visual imagery, of the new atomic theory should follow from its equations and be grounded in the 'uncertainty principle'. What this means is that, unlike in classical physics, in the atomic domain the measurement uncertainties in position and momentum cannot be simultaneously reduced to zero. Rather, the product of these uncertainties is an extremely small but nonzero quantity. In concrete terms: the more precisely the particle's position is measured, the less precisely can its momentum be ascertained, and vice versa.

Heisenberg was able to give his ideas precise mathematical form. It involved the uncertainty, or rather 'indeterminacy' or 'imprecision in knowledge', of simultaneous measurements of position and momentum (for the situations he was considering, momentum p = mass × velocity). Denoting the uncertainty in position as Δx (delta x) and the uncertainty in momentum as Δp (delta p), Heisenberg's uncertainty relation is that the product $\Delta x \Delta p$ is at least $h/(2\pi)$, where h is Planck's constant (6.6×10^{-34} joule-seconds).[18] Its perhaps unfamiliar units aside, although Planck's constant is an extremely small quantity, it is not zero. This is why, according to the uncertainty principle, the more precisely we can measure a particle's position, the less we know about its momentum at the same time. This completely contradicts the common-sense or intuitive idea in Newton's physics that there is no reason at all why, at any given moment, we cannot know to any desired accuracy both where a particle is and how fast its moving. For example, according to Newton, the accuracy with which you know the position of a falling apple should, in principle, have nothing to do with how accurately you know its speed at the same time.

Having demonstrated that discontinuities and a particle representation were essential to any new atomic theory, and that Schrödinger's suggested visual imagery drawn from familiar phenomena was insufficient, at this point Heisenberg chose to deal with Schrödinger's *ad hominem* comments in his third communication of 1926. He did so in a footnote, almost as an afterthought. He recalled Schrödinger writing of the matrix version of quantum mechanics as a theory that is 'frightening, indeed repulsive in its counter-intuitivity and abstractness'. Heisenberg continued with a double-sided compliment to Schrödinger as having formulated a theory that could not be esteemed highly enough because it permitted 'mathematical penetration of the quantum-mechanical laws'. However, Heisenberg continues, in his 'opinion' its 'popular intuitivity' led scientists astray from the 'direct path' for the consideration of physical problems.

By this time it was clear that Schrödinger had no intention of fighting back in print. But privately Schrödinger persisted in his view of the possibility of a visual imagery of waves for elementary particles, of no probabilities entering the picture, and of no quantum jumps. On 4 October 1927 Schrödinger arrived at Bohr's institute in Copenhagen to lecture on his theory. Heisenberg recalled what happened:

> Bohr's discussions with Schrödinger began at the railway station and were continued daily from early morning until late at night. Schrödinger stayed in Bohr's house so that nothing would interrupt the conversations. And, although Bohr was normally most considerate and friendly in his dealing with people, he now struck me as an almost remorseless fanatic, one who was not prepared to make the least concession or grant that he could ever be mistaken. It is hardly possible to convey just how passionate the discussions were, just how deeply rooted the convictions of each, a fact that marked their every utterance.[19]

Discussing various ways in which the electron could make atomic transitions, Schrödinger concluded, 'The whole idea of quantum jumps is sheer fantasy.' Bohr's reply was simply: 'Yes, in what you say, you are completely right. But that doesn't prove there are no quantum jumps. It only proves that we can't visualize them.'[20] One of Schrödinger's final retorts at Bohr was that 'if all this damned quantum jumping were really here to stay, I should be sorry I ever got involved with quantum theory'.[21] By this time the strain had made Schrödinger ill with fever and he had taken to bed. Bohr's wife took meticulous care of him. But Bohr was relentless – sitting on the edge of Schrödinger's bed, he continued to press his argument; 'But you must surely admit that . . .'[22] Schrödinger refused to capitulate. He continued to believe that atomic processes could be visualized with the old imagery, suitably redefined. But Bohr thought otherwise, and had become increasingly interested in Heisenberg's uncertainty principle, which indicated that the equations of quantum mechanics would point the way to an entirely new visual imagery. Physics had come full circle back to the view of Plato, some 2,000 years earlier, in which mathematics would be the guide to what constitutes physical reality.

The Schrödinger equation turned out to have an enormously wide range of applications. This became clear immediately for chemistry when a new branch of research emerged, quantum chemistry, which studies the bonding

of atoms and such complex situations as molecular bonding and chemical reactivity. The earliest triumph of Schrödinger's equation in this area is Walther Heitler and Fritz London's description in 1927 of the bonding of the hydrogen molecule. This sort of problem was, of course, impossible even to approach in the old Bohr theory of the atom. It was based on another of Heisenberg's dazzling discoveries. In 1926 he had deduced the helium atom's spectrum, a problem that had defeated everyone in the old Bohr theory. The dazzling aspect of the discovery is that in quantum theory particles can attract one another by exchanging places extremely rapidly. This exchange phenomenon is at the basis of Heitler and London's theory and would also be central to the first theory of the force that holds the nucleus together, formulated by Heisenberg in 1932.

The Schrödinger equation can also be used to study how chemicals react at a molecular level, the details of which are usually extremely difficult if not impossible to observe experimentally. The wave function of every molecule is very complicated: it has to take into account both the relative positions and the interactions of all the constituent particles. To compute these wave functions from the Schrödinger equation by hand is a virtual impossibility – computers are essential. For this reason, the computation of these wave functions – and the chemists' understanding of chemical processes at a molecular level – has burgeoned since the development of increasingly high-speed computers in the late 1970s. The consequence has been advances in almost all areas of chemistry, from the production of new drugs to the study of the Earth's atmosphere.

The province of the Schrödinger equation is not restricted to the atomic and subatomic domains. It is also needed to explain some extraordinary effects that we see in the large-scale world, notably superconductivity and superfluidity. Superconductors are special materials whose electrical resistance drops suddenly to zero when the temperature falls to below a critical value that is usually below –250 celsius, extremely cold by everyday standards. Such materials have many extraordinary attributes, not least that they all completely expel magnetic fields when they are superconducting. The phenomenon of superfluidity is similarly puzzling. It occurs only in liquid helium at extremely low temperatures, when very strange things happen – it flows practically without viscosity and can even climb up and over the walls of vessels that contain the liquid. The remarkable thing is that both superconductivity and superfluidity can be tackled theoretically by using the Schrödinger equation, applied to the matter's constituent atoms and molecules.

Besides playing an integral role in physics and chemistry, the Schrödinger equation has become an active topic in philosophy. Consider the so-called measurement problem. Whereas in classical physics the interaction between the measurement apparatus with the system under investigation can be ignored, this is not so in quantum theory. For example, consider the following experiment. I want to measure the position of a falling marble, which I can accomplish by, say, photographing it. This process entails that the marble be illuminated and that light be reflected from the falling marble onto a photographic plate. The fact that the marble is being bombarded with light quanta makes pretty much no difference at all to the outcome. In practice the marble's position and its velocity (and so its momentum, too) can be determined simultaneously to any desired degree of accuracy.

But what if the marble is an electron? According to wave mechanics, the falling electron can be anywhere because its wave function is spread out over all of space. The marble, on the other hand, is localized right from the start.[23] Clearly the question 'What is the electron's position?' *really* has no meaning until an actual measurement is carried out, in this case by photographing it. Photographing the electron means illuminating it with at least one light quantum, which becomes part of the measurement system. The interaction of this single light quantum with the electron locates the electron at that moment. This is known as 'collapse of the wave function' because the interaction between the measurement system (light quantum) and system in question (electron) reduces the electron's previously spread-out wave function to a certain well-defined region of space. In other words, of all the possible positions that the electron can have as a wave spread out over all of space, a single one is selected by the measurement process. Therefore, the state of the electron is irreversibly changed from being potentially everywhere to being definitely somewhere. The uncertainty principle informs us that the cost is an enormous uncertainty in the electron's momentum. One of the enduring puzzles of quantum theory concerns what happens to the wave function of an electron (or any other quantum) during a measurement. Before the measurement is made, the electron is in a combination of several quantum states, but the very act of measurement is believed – according to standard quantum lore – to put it in one particular state. What on earth is the underlying mechanism behind this? On this fundamental question, the Schrödinger equation and the other fundamental equations of quantum theory are silent.[24]

There is an interesting photograph of the Nobel Prize winners for 1933 taken at the Stockholm train station. Dirac is to Heisenberg's right and Schrödinger to his left. Dirac and Heisenberg are in formal suits and overcoats. In most photos Heisenberg is either smiling or in some sort of dignified, serious pose, but here he has turned away from Schrödinger with a look almost like disgust. Schrödinger, alone of the three, has a big grin and seems to be having the time of his life. He is in the flamboyant attire of the day: calf-length trousers with bottoms bloused over elastic ends and high socks, casual coat with large fur collar, and his signature bow tie. Another memorable photograph in which the two adversaries are both present is also telling of their bitterly divergent views. It is at the annual summit gathering of physicists – the 1933 Solvay Conference in Brussels. As is the style in these photos, the elder conferees sit while the younger ones stand. In time the younger ones begin to move into the seats. Schrödinger sits and Heisenberg stands almost but not quite directly behind him.

Although many physicists consider quantum theory to be a closed book, there are still fundamental issues that remain unsettled, and most of them are rooted in the Schrödinger equation. Schrödinger wrote on 23 March 1936 to Einstein of his recent meeting with Bohr in London, 'I found it good that they strive in such a friendly way to bring one over to the Bohr–Heisenberg point of view . . . I told Bohr that I'd be happy if he could convince me that everything is in order, and I'd be much more peaceful.'[25] Bohr never could.[26] Instead, he isolated Schrödinger.

The battle lines were quickly and clearly drawn in the struggle between the waves and particles. Things seemed to be going well for a while for Schrödinger's cause. Until, that is, the winter of 1926, when Bohr summoned Heisenberg to Copenhagen to hammer out the meaning of quantum physics. Their deliberations went on for much of the following year. During this time they worked out the so-called Copenhagen interpretation with its emphasis on probabilities, discontinuities and wave-function collapse, all of which were anathema to Schrödinger. But he was no match for them. Schrödinger did not fight either in print or at the famous 1927 Solvay Conference, leaving it to no lesser a figure than Einstein to fly the flag. But Einstein also got nowhere with Bohr and company, despite some ingenious counter-proposals. The 'war' lasted a year. Whereas Schrödinger never made another great discovery before or after the equation that bears his name, Heisenberg had several notable successes before June 1925 and would go on throughout the mid-1930s to do more great work. He would always remain a force to be reckoned with. In the pantheon of twentieth-century physics, Heisenberg is second only to Einstein.

Ironically, although Heisenberg won the battle, and felt he had won the war, Schrödinger's equation is more widely used than Heisenberg's version of atomic physics. This is the case despite the incompatibility of Schrödinger's equation with relativity, which is unimportant for just about every practical application, notably because most of these applications involve quanta that travel at nowhere near the speed of light. On the other hand, Heisenberg's matrix formalism found its role in deeply theoretical areas such as the quantum field theory of fundamental particle physics.

What I have always found so intriguing about the Heisenberg–Schrödinger dispute is that it was fundamentally one of aesthetic choice. Both versions of atomic physics could account, in principle, for all known experimental data about the hydrogen atom and were fundamentally equivalent, in that they gave the same explanations about, for example, the helium atom. Each man defended his view of nature passionately. Bohr's great realization here is that neither man took serious account of the wave–particle duality of light and matter. And here Bohr made a key point: there is a third aesthetic, in which waves and particles are taken together, within a suitable interpretation of Schrödinger's wave function, which was already to hand – namely Born's.

That there are two versions of atomic physics should come as no surprise, because in our world of perceptions things come in pairs, such as particles and waves, yin and yang, black and white, yes and no, love and hate, light and darkness – there are no intrinsic maybes as there are in the atomic world. Yet through abstraction, through emphasis on conception rather than perception, we can move onto a higher plane and appreciate the power of ambiguity. This is generally uncomfortable in our personal lives, in which we strive to resolve ambiguous situations through decisiveness – once again into an 'either/or' mode. As Einstein and Picasso demonstrated in the first decade of the twentieth century, ambiguity is the key to discovering representations of nature that are beyond mere superficial appearances. Direct viewing can deceive, as Einstein discovered in physics and Picasso discovered in art. In Einstein's relativity theory of 1905, time and space are relative, and are interpreted according to how different observers view them. For example, two events that occur at the same time to one observer will not be simultaneous for another observer in relative motion. In Picasso's great work of 1907 *Les Demoiselles d'Avignon*, from which the cubism of Georges Braque and Picasso developed, the painter discovered a way of representing figures so that many possible perspectives appear on the canvas all at once.[27] In their own ways, Schrödinger and Heisenberg carried this adventure of abstraction into the atomic world.

The literary critic William Empson has argued eloquently that the insights of quantum theory could illuminate literature as well.[28] Before switching to literature in 1928 while a student at Cambridge, Empson had read mathematics and was well versed in physics. He developed new interpretations of Shakespeare's works, seeing fit to 'attach the notion of probability to the natural object rather than to the infallibility of the human mind'.[29] Empson advocated renewing the study of literature through the lens of a reality altered by quantum theory. By this he meant that Shakespeare ought not to be analysed in an 'either/or' mode, but the focus should be on ambiguities, that is, a 'both/and' mode, which can bring out hitherto hidden textual meanings. It is possible for a text to have two contradictory meanings at once, as in the wave/particle duality. One of Empson's examples is how to interpret a character as complex as Falstaff. One must accept him as the sum total of apparent opposites, 'as the supreme expression of the cult of mockery as strength and the comic idealization of freedom, yet as both villainous and tragically ill-used'.[30] In Empson's view, the reader ought to 'hold in mind a variety of things [Shakespeare] may have meant, and weigh them . . . according to their probabilities',[31] just as the physicist represents the state of an atom with wave functions.

The concepts of quantum theory, with its deep abstractions, now permeate every aspect of our life. They have required us to rethink a wide range of subjects, transforming our intuitive understanding of nature. Quantum theory is used daily by almost every physicist, yet few of them have ever paused to think about its interpretive subtleties. Like a great work of literature, quantum theory is open to many different interpretations. Most physicists are unaware of this and assume that what they read in the texts of quantum theory texts is catechism. So ingrained has this attitude become that authors no longer state that they are presenting the Copenhagen interpretation, set down during 1926–27 by Bohr and Heisenberg. It has been my experience in teaching the history and philosophy of physics that the more thoughtful physics students are taken by complete surprise and are troubled, having come to expect certainty in textual exposition, instead of ambiguity in interpretation. As the physicist who did more to delve into the foundations of quantum theory than anyone else since Bohr, Einstein and Heisenberg, John Bell, once put it, 'for all practical purposes' quantum physics works well.[32] He forcefully reminded us, however, that we still do not fully understand the Schrödinger equation. As the great intuitive physicist Richard Feynman wrote in his usual pungent style, 'I think I can safely say that nobody understands quantum mechanics.'[33]