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Commentary

Wave vortices 50 years on

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This commentary celebrates the 50th anniversary of the seminal paper by John F. Nye and Michael V. Berry published in 1974 that described topological wave singularities. Originally termed "wave dislocations," they are now known as "wave vortices" and occur in diverse systems, such as tides, quantum matter waves, and most notably, light.

Wave vortices as nodal topological singularities

A striking phenomenon any student of wave physics learns is that when multiple waves are added and destructive interference occurs, it causes the wave amplitude to cancel out. In their 1974 paper "Dislocations in wave trains,"[1] John Nye and Michael Berry described wave interference very generally where waves are reflected by random rough surfaces. They found that in typical wave-interference patterns on the wavelength scale, the amplitude vanishes at points in two dimensions and lines in three. Since these generalized nodes—threads of darkness—disrupt the regular phase pattern of a regular, coherent wave train, Nye and Berry dubbed them "wave dislocations" (see Figure 1A). Nowadays, these structures are more frequently called "wave vortices" (or, depending on the wave's nature, optical vortices, quantum vortices, and so on) on account of the flow of energy around them. In addition to having zero amplitude, these dislocations are topological phase singularities: all 2π phases occur nearby, winding around a positive or negative number of turns. The integer number of phase turns around the dislocation, ℓ, is often called a topological charge and is preserved under small perturbations to the system.

Nye's background made him ideally suited to make this discovery. Seminal experiments with two-dimensional (2D) crystals of "bubble rafts" were performed in the 1940s by W.L. Bragg and Nye, who was a graduate student. This work, later published in volume 2 of the *Feynman Lectures on Physics*, led to the general acceptance of crystal dislocations. Nye's interest in crystals led him to glaciology and thus to a consulting role for the British Antarctic Survey. The randomly reflected ultrasonic waves, which Nye used to demonstrate wave dislocations, had been designed to mimic the random radio waves being used to map the land under the Antarctic ice sheet. Nye had been at the University of Bristol since the 1950s, in an atmosphere highly conducive to developing concepts related to topology. In particular, Sir F. Charles Frank, who contributed significant developments to condensed matter theory, proposed the idea of "disclinations"—topological defect lines—in liquid crystals in 1956. In Bristol in 1959, Yakir Aharonov and David Bohm discovered the Aharonov-Bohm effect[4]: the topological effect by which a magnetic solenoid line enclosed within a quantum particle's loop trajectory causes an observable shift to the quantum phase. Michael Berry joined the University of Bristol in the mid-1960s, and in 1984, he discovered the closely related geometric (or Berry) phase appearing in cyclic quantum evolutions.[5] By the late 1960s, he was working on caustics, and by the late 1970s, Nye and Berry were collaborating on the description of wave caustics using catastrophe theory. They also worked together on dislocations in wave fields, leading to their classification and simple mathematical models.

Nye and Berry acknowledged wave dislocations in previous studies of various physical waves. Notably, examples of phase singularities are known from the 19th century as "amphidromic points" in ocean tidal waves (discovered by William Whewell in 1833) (Figure 1B). The tidal amplitude vanishes at these points, whereas the high-tide peak circulates around the amphidromic peak with the tidal-wave period. Furthermore, wave dislocations play a crucial role in the fundamental quantum-mechanical construction of Paul Dirac's hypothetical magnetic monopole (1931). Namely, the electron wave function around the monopole inevitably has a dislocation line, which ends with the monopole. In the 1940s, several independent authors, including Arnold Sommerfeld, Hans Wolter, Werner Braunbek, and Trevor Pearcey, described energy-flow vortices in electromagnetic and optical waves[6] (some, at least, influenced by studies on radio wave scattering during World War II).

Wave topology branches out

Nye and Berry's paper was adopted, and over the next two decades, wave dislocations were well studied as generic features of disrupted and chaotic wave patterns, including optical speckle patterns and chaotic quantum wave functions. Moreover, inspired in part by analogous structures in liquid crystals, Nye and co-workers extended the idea of topological singularities to vector waves. They studied light with position-dependent

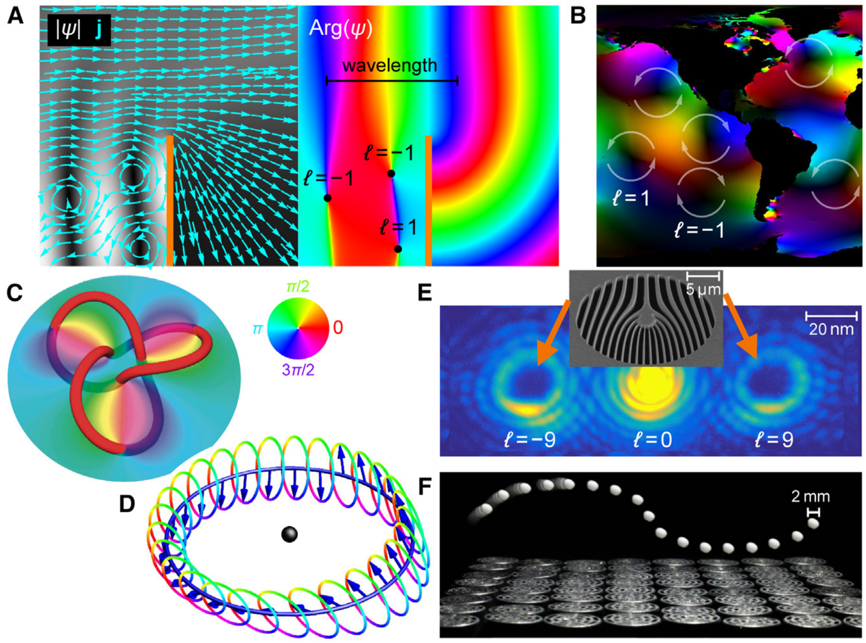

Figure 1. Examples of "wave dislocations" (phase singularities or wave vortices) and other topological structures in various kinds of wavefields

(A) Diffraction of a plane wave incident from left to right on a half-plane obstacle (described by Braunbek and Laukien in 1952, reprinted in several subsequent textbooks). Left panel: the wave amplitude $|\psi(x,y)|$ (grayscale) and the wave current (or energy flux) $\mathbf{j} = \mathrm{Im}(\psi^* \nabla \psi)$ (cyan). Right panel: the wave phase Arg $[\psi(x,y)]$ coded via hue colors. Phase singularities (black dots, with their topological charges ℓ) appear on subwavelength scale before the obstacle, and the wave current swirls around these.

(B) Phase singularities, known as amphidromic points, and phase vortices around these on the map of M2 ocean tides. The tidal amplitude and phase are coded by brightness and color, respectively (HAMTIDE model data, E.E. Taguchi, D. Stammer, and W. Zahel [2014]. *Geophys. Res. Oceans* 119, 4573–4592; this plot is courtesy of Kateryna Domina).

(C and D) Examples of more sophisticated 3D topological wave structures: (C) a trefoil knot of the phase-singularity line (reproduced from M. Dennis, R. King, B. Jack, et al. [2010]. Isolated optical vortex knots. *Nature Phys*. 6, 118–121. https://doi.org/10.1038/nphys1504) and (D) Möbius band formed by the orientations (blue vectors) of polarization ellipses (phase-colored ellipses) surrounding the C-point with a purely circular polarization (black dot) (reproduced from K.Y. Bliokh, M.A. Alonso, D. Sugic, M. Perrin, F. Nori, E. Brasselet [2021]. Polarization singularities and Möbius strips in sound and water-surface waves. *Physics of Fluids* 33, 077122. https://doi.org/10.1063/5.0056333).

(E) Quantum free-electron vortices generated in a transmission electron microscope (TEM) using a hologram with fork-like dislocation (gray). The vortices with topological charges $\ell = \pm 9$ are produced in the first diffraction orders (Bliokh et al.[2]).

(F) Holographic 3D acoustic-vortex-based tweezers manipulating polystyrene particles in the air (Marzo et al.[3]).

polarization patterns, identifying C points and C lines where polarization is circular, and the ellipse's azimuth is singular.[7] Phase and polarization singularities were later emphasized as a "topological skeleton" for a complex texture of interfering waves.

Interest in optical vortices became turbocharged after 1992 when the Leiden group of Han Woerdman, working with Les Allen, discovered structured laser-beam eigenmodes of the orbital angular momentum (OAM) operator (with respect to the beam axis).[8] At a similar time, Marat Soskin's group in Kyiv showed[9] that a dislocated hologram pattern can plant an optical vortex in a laser beam. The OAM eigenmodes, now known as "vortex beams," contain circularly symmetric phase singularities on the beam axis (where helicoidal wavefronts meet—analogous to crystalline screw dislocations), and the topological charge ℓ becomes the OAM quantum number. This remarkable interconnection of the topological and dynamical properties resulted in rapid and fruitful development of the vortex beams research and applications.

In the following decades, the original ideas of wave dislocations and vortex beams evolved into extended areas of research with diverse implications in different fields. On the one hand, the concept of universal topological structures in complex wavefields was extended to more complex objects, such as knotted singularities (Figure 1C), polarization Möbius bands[10] (Figure 1D), and, most recently, skyrmionic textures—i.e., linear-vector-wave configurations analogous to 2D and three-dimensional (3D) winding textures in high-energy and condensed-matter systems. In contrast to the localized point-like or line-like singularities, these textures are extended, continuous objects occupying a certain area of space or plane. All these findings serve the general idea to characterize complex wave fields by their topological features. Indeed, the usual wave characteristics, such as the local amplitude, phase, and polarization, can considerably vary under inevitable small perturbations in complex systems, while the topological structures and their topological numbers are robust with respect to such perturbations.

Over a similar period since the mid-1970s, a somewhat parallel topological approach reshaped condensed-matter physics. With a focus on topological structures in momentum space rather than real space, these studies began with the discovery of the quantum Hall effect, leading to the modern classification of a rich variety of topological insulators and superconductors. This difference between "optical" and "condensed matter" approaches is natural; in optical interference, real-space complex structured fields are directly measurable, whereas in complex materials, the internal wave fields are inaccessible and the measurable Fourier spectra become the main objects of interest. Notably, topological properties of condensed-matter systems are essentially underpinned by Berry phases in momentum space, and these ideas involving topological phases and quantized vortices were the basis of the 2016 Nobel Prize in Physics, awarded to David Thouless, Michael Kosterlitz, and Duncan Haldane.

Vortex beams at work

Optical vortex beams have found numerous applications and have been exported to other areas of wave physics.

Optical vortices have contributed to almost every area of classical optics, including nonlinear optics with a wide class of vortex solitons,[11] laser optics with vortex microlasers, and even astronomy with vortex coronography.[12] Combined with specific fluorescent molecules, vortex beams are a key component of super-resolved stimulated emission depletion (STED) microscopy, earning Stefan Hell a share of the 2014 Nobel Prize in Chemistry.

The use of vortex beams revolutionized optical[13] (and later acoustic[3]) tweezers and the manipulation of small particles. This field began with Arthur Ashkin's works using usual Gaussian-like laser beams (leading to his share of the 2018 Nobel Prize in Physics). The use of vortex beams allowed effective rotational manipulation of particles and entrapment of particles near the zero-amplitude (dark-field) zones, reducing thermal effects. In the past two decades, the use of vortex and other phase-structured optical and acoustic beams resulted in the development of holographic tweezers (Figure 1F). Such tweezers allow simultaneous 3D manipulation of a large number of particles, from single atoms to biological cells and microorganisms, with futuristic applications, such as volumetric displays based on the 3D arrays of trapped particles.

Furthermore, optical vortex beams have found remarkable implications in quantum optics and information transfer. In 2001, Anton Zeilinger's group in Vienna[14] successfully measured quantum entanglement of the OAM (vortex) states of photons, extending the optical Hilbert space accessible to fundamental quantum experiments from two (via polarization) to an arbitrary number of dimensions (the mode's topological vortex strength). Zeilinger's work on optics and quantum information was honored in his share of the 2022 Nobel Prize in Physics. The same idea of the OAM mode multiplexing was applied to increase the information transfer via electromagnetic and optical signals in free space and optical fibers.[15]

Importantly, vortex beams and other topological wave structures have transcended the boundaries of optics and provided a useful toolbox across all areas of wave physics, both classical and quantum. OAM vortex beams, as well as a variety of topological wave structures, have been generated in acoustics, plasmonics, and even water waves. In the quantum domain, vortex beams have been produced and found applications in electron microscopy[2] (Figures 1E), as well as in neutron and atomic waves.

Dark threads have a bright future

Optical vortices and topologically structured light have been crucial to our contemporary understanding and ability to manipulate light and other waves. We have briefly reviewed this incredible transformation over the past 50 years, and how optical and acoustic vortices provide the means to manipulate bio- and nanoparticles, study and control quantum information, and improve optical imaging from microscopes to telescopes. Originally identified with defects in crystals, optical and acoustic vortices now provide a basis for topological analogies in a gamut of wave systems of different nature. As optical technologies move into the nanoscale regime, the fine structure of light fields becomes increasingly significant, suggesting the dark threads in light beams have a bright future.

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DECLARATION OF INTERESTS

The authors declare no competing interests.

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